Position/Force control using sliding mode with H_∞ attenuator to reduce rebounds in a mechanical system with a position constraint

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Abstract. This paper focuses on the problem of the control of a three degrees-of-freedom mechanical system, subject to constraints on the position, dry friction Dahl type and external disturbances. It is proposed a controller using sliding mode with an H_∞ attenuator to solve a position/force regulation problem. It is proved using Lyapunov tools that the nonlinear system has a local equilibrium asymptotically stable and achieves zero steady-state position error even in the presence of certain disturbances and dynamical friction. As well is given a parameter tuning that could reduce the number of rebounds between the end-efector and the position constraint. Furthermore, the controller attenuates other external perturbations and model discrepancies. The results obtained are illustrated with experiments.

1 Introduction

This paper focuses on the regulation problem for a mechanical system with a position constraint. The methodology of sliding mode with H_∞ attenuator is applied to solve the problem in question. Some recently references using this methodology can be found in [1–3]. Basically is a sliding mode controller that involves an H_∞ control design on the sliding surface. The purpose of this design is the elimination of disturbances and parametric uncertainties through the sliding mode control, otherwise, the H_∞ design will attenuate the parametric uncertainties and disturbances, by this way the trajectories will remain bounded around the reference.

The methodology to solve the regulation problem in a mechanical system under unilateral constraints was previously addressed by [4]. In [5] was constructed a PID controller type for robots under constraints, where a $\mathcal{H}_2/\mathcal{H}_\infty$ control is proposed to attenuate the influence of disturbances and uncertainties.

In [6] was designed an integral control using nonlinear H_∞ control and sliding modes for a permanent magnet synchronous motor. Another example of this type of controllers can be found in [2] where a sliding mode control with an H_∞ approach is utilized for output control. At the same time in [7], it is presented a mixed H_∞-sliding mode controller used to control a magnetic levitation system.

Furthermore [8] used an strategy of H_∞ and sliding modes applied to a current control problem in a switched converter. Finally [9] analyzed and designed an integral sliding mode controller combined with H_∞ to control systems with unmatched perturbations.

The present paper considers that the dynamical system is nonlinear. Additionally, the motion of the system is affected by unknown disturbances, and the available state measurements are incomplete and imperfect.

To prove the stability of the controlled mechanical system we use quadratic functions; some references can be found in [10–13]. These quadratic functions allow us to ensure that the trajectories converge asymptotically to the desired position, and prove the convergence to the sliding surface in finite time.

In this study we combine the robustness properties of sliding mode with H_∞ control to design a controller which is capable to handle the above mentioned factors and thereby yielding a good performance on real systems. Experiments confirm the validity of the theoretical analysis.

The paper is organized as follows: In Section 2 it is defined the problem statement. In Section 3, the design procedure is considered. The develop of the sliding mode with H_∞ attenuator control is addressed in Section 4. Stability analysis is approached in Section 5. In Section 6 addresses the issue of H_∞ synthesis. Experiments are offered in Section 7. Finally in Section 8 some conclusions are discussed.

2 Problem Statement

The main concern of this work is the regulation control design, and its stability analysis, of a mechanical system subject to a position constraint (see Figure 1). This is a system, described by a lagrangian model. It can display an important dynamical behaviour like rebounds, due to collisions with the constraint, which may risk the integrity of a mechanical device. Hence, we design the controller with the aim, besides of having a good regulation, to reduce the presence of this phenomenon.

The equations of motion of the open-loop constrained mechanical system can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau + \tau_c(q) + w(t) \tag{1}$$

where $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbf{R}^3$ represent the displacement, velocity and acceleration of the rotational links of the mechanical system; $M(q) \in \mathbf{R}^{3\times 3}$ is the inertia matrix, which is symmetric and positive definite for all $q \in \mathbf{R}^3$; $C(q,\dot{q})\dot{q}$ is the vector of centripetal and Coriolis forces; $G(q) \in \mathbf{R}^3$ is the vector of gravitational forces; $\tau_c(q) \in \mathbf{R}^3$ is the torque generated by the spring by making contact with constraint; $\tau \in \mathbf{R}^3$ are the control inputs, and $w(t) = [w_1(t), w_2(t), w_3(t)]^T \in \mathbf{R}^3$ are unknown external disturbances. $F(\dot{q}) \in \mathbf{R}^3$ is the vector of frictional forces, which are represented as a combination

$$F_i = \sigma_{0i}\dot{q}_i + F_{di}, \quad i = 1,2,3 \tag{2}$$

of viscous friction $\sigma_{0i}\dot{q}_i$ and Dahl friction F_{di} which is governed by the following dynamic model:

$$\dot{F}_{di} = \sigma_{1i}\dot{q}_i - \sigma_{1i}|\dot{q}_i|\frac{F_{di}}{F_{ci}} + w_{2i}, \tag{3}$$

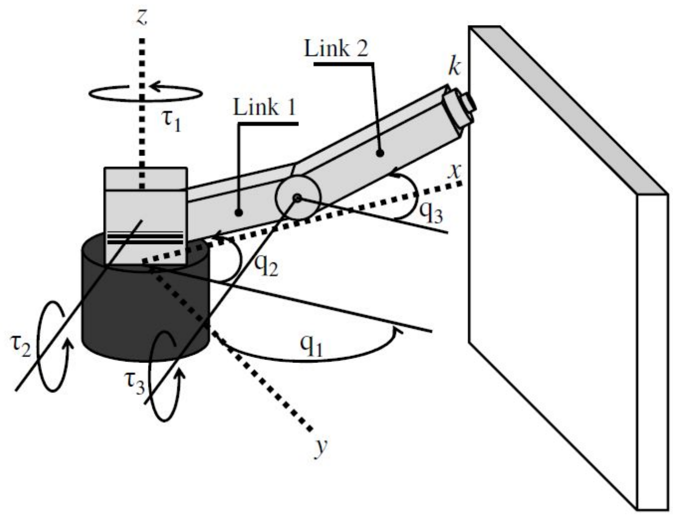

Fig. 1. Pegasus robot with three degrees-of-freedom and position constraint

where $\sigma_{0i} > 0$, $\sigma_{1i} > 0$, and $F_{ci} > 0$ are the viscous friction coefficient, stiffness coefficient and Coulomb friction level, respectively, corresponding to the ith manipulator joint; w_{2i} is an external disturbance which is involved to account for inadequacies of the frictional model.

The Dahl model (3) describes the spring-like behaviour during stiction and is essentially Coulomb friction with a lag in the change of the friction force when the motion direction is changed. Since the Coulomb friction is only a function of the displacement and the sign of the velocity, this dynamic model appears nonsmooth (see for details).

Clearly, the above component-wise relations can be rewritten in the vector form

$$F = \sigma_0 \dot{q} + F_d, \tag{4}$$

$$\dot{F}_d = \sigma_1 \dot{q} - \sigma_1 \text{diag}\{|\dot{q}_i|\} F_c^{-1} F_d + w_2, \tag{5}$$

where $F = \text{col}\{F_i\}$, $F_d = \text{col}\{F_{di}\}$, $x = \text{col}\{q_i\}$, $\sigma_0 = \text{diag}\{\sigma_{0i}\}$, $\sigma_1 = \text{diag}\{\sigma_{1i}\}$, $F_c = \text{diag}\{F_{ci}\}$, $w_2 = \text{col}\{w_{2i}\}$, the notations diag and col are used to denote a diagonal matrix and a column vector, respectively. The Euclidean position between the origin of the inertial frame of the robot and the constraint is given by x_0, since the constraint is located along the x axis, the position of the end effector of the robot with respect to x axis is given by x_r, at the same time the position of the end effector with respect to y axis is given by y_r, and it is z_r with respect to z, which are denoted as $R(t) = [x_r(t), y_r(t), z_r]^T \in \mathbf{R}^3$. A spring is added at the end effector tip, which is punctual with a stiffness coefficient k, which acts as a force sensor. One way to represent the force generated by the spring is using the Hooke law $F = kx_r$.

An impact is generated between the end effector of the robot and the constraint when $x_r \geq x_0$ where $x_r = [l_1 \cos(q_2) + l_2 \cos(q_3)] \cos(q_1)$. The impact generates forces of equal magnitude and opposite directions between the robot and the constraint. Specifically, the impact force acting in the spring $F_c \in \mathbf{R}$ is defined as follows

$$F_c = \frac{k}{2} \left(x_r - x_0 + |x_r - x_0| \right). \tag{6}$$

The impact force acting on the links of the robot produce a torque denoted by $\tau_c(x_r, q) \in \mathbf{R}^3$, given by

$$\tau_c = -F_c \begin{bmatrix} [l_1 \cos(q_2) + l_2 \cos(q_3)] \cos(q_1) \\ [l_1 \cos(q_2) + l_2 \cos(q_3)] \sin(q_1) \\ l_1 \sin(q_2) + l_2 \sin(q_3) \end{bmatrix}. \tag{7}$$

The above term completes the model (1) for the three degrees-of-freedom Pegasus robot constrained on the position.

3 Design Procedure

The control objective is to find a control law $\tau \in \mathbf{R}^3$, which depends on the desired force at the spring F_d (through the desired position x_{d1} along the x axis), the joint positions (q_1, q_2, q_3), the reference position x_0, the joint velocities $(\dot{q}_1, \dot{q}_2, \dot{q}_3)$, and the joint desired positions (q_{d1}, q_{d2}, q_{d3}) such that the system in closed loop satisfies

$$\lim_{t\to\infty} |q_1(t) - q_{d1}| = 0, \quad \lim_{t\to\infty} |q_2(t) - q_{d2}| = 0, \quad \lim_{t\to\infty} |q_3(t) - q_{d3}| = 0 \tag{8}$$

in spite of the upper bounded disturbance $\sup_t \|w_1(t)\| \leq N \in \mathbf{R}^3$, where the H_∞ control attenuates the influence of another external disturbances w_0, $w_2 \in \mathbf{R}^3$. Given the fact that $F_d = k(x_{d1} - x_0)$, with $F_d \geq 0$ and $x_{d1} = [l_1 \cos(q_{d2}) + l_2 \cos(q_{d3})] \cos(q_{d3})$, by substituting x_{d1} into F_d it is obtained q_{d2}

$$q_{d2} = \arccos \left(\frac{F_d}{l_1 k \cos(q_{d1})} + \frac{x_0}{l_1 \cos(q_{d1})} - \frac{l_2 \cos(q_{d3})}{l_1} \right) \tag{9}$$

Can be shift the equilibrium point of (1) to the origin by introducing the transformation based on the following

$$x_1 = \int_0^t x_2(t) dt, \; x_2 = [q_1 - q_{d1}, q_2 - q_{d2}]^T, \; x_3 = [\dot{q}_1, \dot{q}_2]^T, \; x_4 = [F_{d1}, F_{d2}]^T. \tag{10}$$

hence the state space equations are as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= M^{-1}(x_2 + q_d)[-C(x_2 + q_d, x_3)x_3 - G(x_2 + q_d) \\ &\quad - \sigma_0 x_3 - x_4 + \tau_c(x_2 + q_d) + u + w_1] \\ \dot{x}_4 &= \sigma_1 x_3 - \sigma_1 \text{diag}\{|x_{3i}|\} F_c^{-1} x_4 + w_2. \end{aligned} \tag{11}$$

where diag$\{q_d\}$, diag$\{\sigma_0\}$, diag$\{\sigma_1\}$, diag$\{w_1\}$, diag$\{w_2\}$, and diag$\{F_c\} \in \mathbf{R}^{3\times 3}$.

4 Sliding Mode Control using H_∞ Attenuator

Let us consider the following sliding surface

$$s = \nu x_1 + \mu x_2 + x_3 - \int_0^t u_\infty dt \tag{12}$$

where u_∞ is an H_∞ control which operates locally around the equilibrium point of system (11), also, the sliding surface (12) is a dynamical variable.

The control law which ensures that trajectories reach the sliding manifold is given by

$$\begin{aligned} u = C(x_2 + q_d, x_3)x_3 + \sigma_0 x_3 + x_4 - \tau_c(x_2 + q_d) + G(x_2 + q_d) \\ -M(x_2 + q_d)\left[x_2 + x_3 - u_\infty + \lambda s + \beta \text{sign}(s)\right]. \end{aligned} \tag{13}$$

The proposed control law will be acting at all time $t \geq 0$, that is, when the system is in free or in constrained motion (in contact with the constraint). The parameters diag$\{\lambda\}$ and diag$\{\beta\} \in \mathbf{R}^{3\times3}$ have positive values which will be tuned to ensure the motion of the trajectories be driven toward the sliding surface.

Due the sliding surface (12)is a dynamical variable, it will be added as another state, this leads to the extended systems

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_2 - x_3 - \lambda s - \beta \text{sign}(s) + M^{-1}(x_2 + q_d)w_1 \\ \dot{x}_4 &= \sigma_1 x_3 - \sigma_1 \text{diag}\{|x_{3i}|\}F_c^{-1}x_4 + w_2 \\ \dot{s} &= \nu x_2 + \mu x_3 + M^{-1}(x_2 + q_d)[-C(x_2 + q_d, x_3)x_3 - \sigma_0 x_3 - x_4 \\ &\qquad + \tau_c(x_2 + q_d) - G(x_2 + q_d) + u + w_1] - u_\infty. \end{aligned} \tag{14}$$

5 Stability Analysis

Theorem 1 *Let the system (14) through the controller governed by (12), (13), considering the condition* $\sup_t |w_{1i}(t)| \leq N_i$ *for all time and a constant value* $N_i > 0$ *and* $w_{2i}(t) = 0$*, with* $i = 1, 2, 3$*. Then the trajectory* q *in system (14) is asymptotically stable.*

Proof. from substituting the control law (13), the closed-loop system takes the form

$$\begin{aligned} s^T \dot{s} &= s^T \left(-\lambda s - \beta \frac{s}{\|s\|} + M^{-1}(x_2 + q_d) \sum_{i=1,2,3} N_i \right) \\ &\leq -\lambda \|s\|^2 - \left(\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}(x_2 + q_d)\} \sum_{i=1,2,3} N_i \right) \|s\|. \end{aligned}$$

Can be conclude the existence of sliding modes on the surface $s = x_1 + x_2 + x_3 - \int_0^t u_\infty dt$ while the condition $\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}x_2 + q_d)\} \sum_{i=1,2,3} N_i > 0$ remains valid. Also, we can demonstrate finite time convergence of the trajectories to the surface $s = 0$ using the quadratic functionrática

$$V(s) = s^T s. \tag{15}$$

and compute its time derivative along the solutions of (14),

$$\begin{aligned}
\dot{V}(s(t)) &\leq -2s^T \lambda s - 2\left(\beta - M^{-1}(x_2 + q_d) \sum_{i=1,2,3} N_i\right) \|s\| \\
&\leq -2\left(\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}(x_2 + q_d)\} \sum_{i=1,2,3} N_i\right) \|s\| \\
&= -2\left(\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}(x_2 + q_d)\} \sum_{i=1,2,3} N_i\right) \sqrt{V(s(t))}.
\end{aligned} \tag{16}$$

From (16) it follows that

$$V(t) = 0 \quad \text{para} \quad t \geq t_0 + \frac{\sqrt{V(t_0)}}{\left(\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}(x_2+q_d)\} \sum_{i=1,2,3} N_i\right)} = t_f. \tag{17}$$

Hence, $V(t)$ converges to zero in finite time and, in consequence, a motion along the manifold $s = [0, 0, 0]^T$ occurs in the discontinuous system (14). Thus, in the following developments, it will be assumed that system (14) is in sliding mode, so $s = \dot{s} = 0$ for $t \geq t_f$. From (12) it is shown that the dynamics of system (14) once on sliding mode, are described by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -\nu x_2 - \mu x_3 + u_\infty \\
\dot{x}_4 &= \sigma_1 x_3 - \sigma_1 \text{diag}\{|x_{3i}|\} F_c^{-1} x_4 + w_2.
\end{aligned} \tag{18}$$

Lemma 1 *Let the system (18), considering the input* $u_\infty = 0$. *Then trajectories* (x_2, x_3) *are asymptotically stable.*

$$V(x_2, x_3) = (x_2 + x_3)^T (x_2 + x_3) + 2x_2^T x_2 + x_3^T x_3 \tag{19}$$

where $V(x_2, x_3)$ is a positive definite and also radially unbounded function. The time derivative of $V(x_2, x_3)$ along the trajectories of (18) with the input $u_\infty = 0$ is given by

$$\begin{aligned}
\dot{V}(x_2, x_3) &= 6x_2^T \dot{x}_2 + 2x_2^T \dot{x}_3 + 2x_3^T \dot{x}_2 + 4x_3^T \dot{x}_3 \\
&= -2x_2^T \nu x_2 - 4x_3^T \mu x_3 + 6x_2^T x_3 - 2x_2^T \mu x_3 - 4x_2^T \nu x_3 + 2x_3^T x_3 \\
&= -2x_2^T \nu x_2 - 4x_3^T \mu x_3 + x_2^T (6 - 2\mu - 4\nu) x_3 + 2x_3^T x_3 < 0.
\end{aligned} \tag{20}$$

By choosing the constants of the main diagonal of matrix $(\text{diag}\{6\} - \text{diag}\{2\mu\} - \text{diag}\{4\nu\})$ making it a zero matrix in compliance with $\text{diag}\{\mu\} > 1/2 \in \mathbf{R}^{2\times 2}$, and $\text{diag}\{\nu\} > 0 \in \mathbf{R}^{2\times 2}$, can be assured while the system remains in $s = 0$, that the trajectories (x_2, x_3) of the system (18) using $u_\infty = 0$ converge to zero at $t \to \infty$.

Thus, the regulation problem for x_2 in the deviation system (18) can formally be stated as a nonlinear H_∞-control problem.

In the sequel, the investigation will be confined to the H_∞ position regulation problem, where

1. The output to be controlled is given by

$$z = \rho \begin{bmatrix} 0 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_\infty \tag{21}$$

   with a positive weight coefficient ρ.
2. The position x_2 is the available measurement and are corrupted by the error vector $w_0(t) \in \mathbf{R}^3$.

$$y = x_2 + w_0, \tag{22}$$

The H_∞ control problem in question is thus stated as follows. Given the system representation (18)-(22) and a real number $\gamma > 0$, it is required to find (if any) a causal dynamic feedback controller

$$u_\infty = K(\xi) \tag{23}$$

with internal state $\xi \in \mathbf{R}^1 2$ such that the undisturbed closed-loop state x_2 is uniformly asymptotically stable around the origin and its $\mathfrak{L}_2$ gain is locally less than γ, i.e., inequality

$$\int_0^T \|z(t)\|^2 dt < \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{24}$$

is satisfied for all $T > 0$ and all piecewise continuous functions $w(t) = [w_0(t), w_1(t), w_2(t)]^T$ for which the corresponding state trajectory of the closed-loop system (18), initialized at the origin, remains in some neighbourhood of this point.

6 H_∞ Synthesis

The above H_∞ control problem in question is the nonlinear H_∞ control problem for nonsmooth systems, modelled by equations of the form

$$\begin{aligned} \dot{x} &= f_1(x) + f_2(x) + g_1(x)w + g_2(x)u \\ z &= h_1(x) + k_{12}(x)u \\ y &= h_2(x) + k_{21}(x)w \end{aligned} \tag{25}$$

where $x \in \mathbf{R}^n$ is the state space vector, $u \in \mathbf{R}^m$ is the control input, $w \in \mathbf{R}^r$ are unknown disturbances, $z \in \mathbf{R}^l$ is the output to be controlled, $y \in \mathbf{R}^p$ are the measurements available in the system. Adapting (18) to the form of (25) leads to

$$f_1(x) = \begin{bmatrix} x_2 \\ x_3 \\ -x_2 - x_3 \\ \sigma_1 x_3 \end{bmatrix}, \qquad f_2(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\sigma_1 \mathrm{diag}\{|x_{3i}|\} F_c^{-1} x_4 \end{bmatrix}, \tag{26}$$

$$g_1(x) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & M^{-1}(x_2+q_d) & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0_{3\times3} \\ 0_{3\times3} \\ I_{3\times3} \\ 0_{3\times3} \end{bmatrix}, \tag{27}$$

$$\begin{aligned} h_1(x) &= \rho \begin{bmatrix} 0_{3\times1} \\ x_2 \end{bmatrix}, \quad h_2(x) = x_2 + q_d, \\ k_{12}(x) &= \begin{bmatrix} I_{3\times3} \\ 0_{3\times3} \end{bmatrix}, \; k_{21}(x) = \begin{bmatrix} I_{3\times3} \; 0_{3\times6} \end{bmatrix} \end{aligned} \tag{28}$$

6.1 Local solution to the H_∞ problem

The following local analysis involve the linear H_∞ control problem for the following system

$$\begin{aligned} \dot{x} &= A_1x + B_1w + B_2u \\ z &= C_1x + D_{12}u \\ y &= C_2x + D_{21}w \end{aligned} \tag{29}$$

where

$$\begin{aligned} A_1 &= \tfrac{\partial f_1(0)}{\partial x} + \tfrac{\partial f_2(0)}{\partial x}, \quad B_1 = g_1(0) \quad B_2 = g_2(0) \\ C_1 &= \tfrac{\partial h_1(0)}{\partial x}, \qquad D_{12} = K_{12}(0) \\ C_2 &= \tfrac{\partial h_2(0)}{\partial x}, \qquad D_{21} = K_{21}(0). \end{aligned} \tag{30}$$

The system (18) must fulfill the stabilizability and detectability conditions around u, and y, respectively. Under these assumptions, the following conditions are necessary and sufficient for a solution of the linear problem to exist (see [14]).

A1 There exists a bounded positive semidefinite symmetric solution of

$$PA_1 + A_1^TP + C_1^TC_1 + P\left[\frac{1}{\gamma^2}B_1B_1^T - B_2B_2^T\right]P = 0 \tag{31}$$

such that the matrix $\left[A_1 - \left(B_2B_2^T - \gamma^{-2}B_1B_1^T\right)P\right]$ has all eigenvalues with negative real part.

A2 There exists a bounded positive semidefinite symmetric solution of

$$AZ + ZA^T + B_1^TB_1 + Z\left[\frac{1}{\gamma^2}PB_2B_2^TP - C_2C_2^T\right]Z = 0 \tag{32}$$

where $A = A_1+(1/\gamma^2)B_1B_1^TP$, such the matrix $\left[A - Z(C_2^TC_2 - \gamma^{-2}PB_2B_2^TP)\right]$ has all eigenvalues with negative real part. The above equations (31) and (32) are known as Riccati equations.

By the bounded real lemma [15], conditions A1 and A2 ensure that there exists a positive constant ϵ_0 such that the system of the perturbed Riccati equations

$$P_\varepsilon A_1 + A_1^T P_\varepsilon + C_1^T C_1 + P_\varepsilon \left[\frac{1}{\gamma^2} B_1 B_1^T - B_2 B_2^T\right] P_\varepsilon + \varepsilon I = 0 \qquad (33)$$

$$A_\varepsilon Z_\varepsilon + Z_\varepsilon A_\varepsilon^T + B_1 B_1^T + Z_\varepsilon \left[\frac{1}{\gamma^2} P_\varepsilon B_2 B_2^T P_\varepsilon - C_2 C_2^T\right] Z_\varepsilon + \varepsilon I = 0 \qquad (34)$$

has a unique positive definite symmetric solution $(P_\varepsilon, Z_\varepsilon)$ for each $\varepsilon \in (0, \varepsilon_0)$ where $A_\varepsilon = A_1 + \left(1/\gamma^2\right) B_1 B_1^T P_\varepsilon$.

Equations (33) and (34) are subsequently utilized to derive a local solution of the H_∞ -control problem as in (25). Let conditions A1 and A2 hold be satisfied, and let $(P_\varepsilon, Z_\varepsilon)$ be the corresponding positive definite solution of (33) and (34) under some $\varepsilon > 0$. Then the output feedback

$$\dot{\xi} = f_1(\xi) + f_2(\xi) + \left[\frac{1}{\gamma^2} g_1(\xi) g_1^T(\xi) - g_2(\xi) g_2^T(\xi)\right] P_\varepsilon \xi + Z_\varepsilon C_2^T \left[y - h_2(\xi)\right] \quad (35)$$

$$u_\infty = -B_2^T(\xi) P_\varepsilon \xi \qquad (36)$$

is a local solution of the H_∞-control problem.

7 Experiments

Performance issues and robustness properties of the proposed sliding mode controller with H_∞ attenuator (13) have been tested in the three degrees-of-freedom platform called Pegasus robot as in Figure 2. Since only the states $[q_1, q_2, q_3]^T$ measurements are available, the H_∞ filter (35) was applied to have access to the remaining states.

The experiments were carried out using the Pegasus robot, simulink from MatLab® and the data acquisition board SENSORAY 626 to be used as interphase between the computer and the robot, as well the force sensor utilized was the FC2231 from Measurement Specialties^{TM} with a measurement range from 0-50 Lbf. The parameters of the Pegasus robot made by Amatrol are shown in Table 1. Initial conditions, controller gains and external disturbances are displayed in Table 2.

8 Conclusions

It was developed a fully practical framework for sliding mode control involving H_∞ control methodology. The afore mentioned design procedure has been shown to be eminently suited to solving a position/force regulation problem for a mechanical system with friction and a position constraint. To facilitate exposition, the friction model chosen for treatment has been confined to the Dahl model augmented with viscous friction. The sliding mode-H_∞ output regulation synthesis proposed is suited to globally solve the regulation problem when the inequality $\lambda_{min}\{\beta\} - \lambda_{max}\{M^{-1}x_2 + q_d)\} \sum_{i=1,2,3} N_i > 0$ is satisfied, even in the presence of disturbances $\sup_t |w_{1i}(t)| \leq N_i$ for all time and a

Table 1: Pegasus robot parameters.

Notation	Description	Value	Units
l_1	Length of the link 1	0.297	m
l_2	Length of the link 2	0.297	m
m_1	Mass of the link 1	0.38	kg
m_2	Mass of the link 2	0.34	kg
I_1	Inertia 1	0.000243	kg m^2
I_2	Inertia 2	0.000068	kg m^2
I_3	Inertia 2	0.000015	kg m^2
g	Gravity	9.80665	m/s^2
l_{c1}	Length to the centre of mass: Link 1	0.1485	m
l_{c2}	Length to the centre of mass: Link 2	0.1485	m

Table 2: Initial conditions, controller gains and external disturbances

Notation	Description	Value	Units
$q_1(0)$	Joint position 1	22.5	grad
$\dot{q}_1(0)$	Joint velocity 1	0	grad/s
$q_3(0)$	Joint position 3	45	grad
$\dot{q}_3(0)$	Joint velocity 3	0	rad/s
$s(0)$	Sliding motion	[0,0,0]	
λ	Controller gains	diag{40,30,30}	$1/s^2$
μ	Controller gains	diag{7,9,9}	$1/(m.s)$
β	Controller gains	diag{1,1,1}	N.m
γ	Controller gains	diag{9,16,16}	$1/(m.s^2)$
F_d	Desired force at the end effector	25	N
q_{d1}	Desired joint position 1	0	rad
q_{d3}	Desired joint position 3	0	rad
w_1	Disturbance set on link 1	$0.1\sin(t)$	N.m
w_2	Disturbance set on link 2	$0.1\cos(t)$	N.m
w_3	Disturbance set on link 3	$0.1\cos(t)$	N.m

Fig. 2. Pegasus robot configured to have a position constraint.

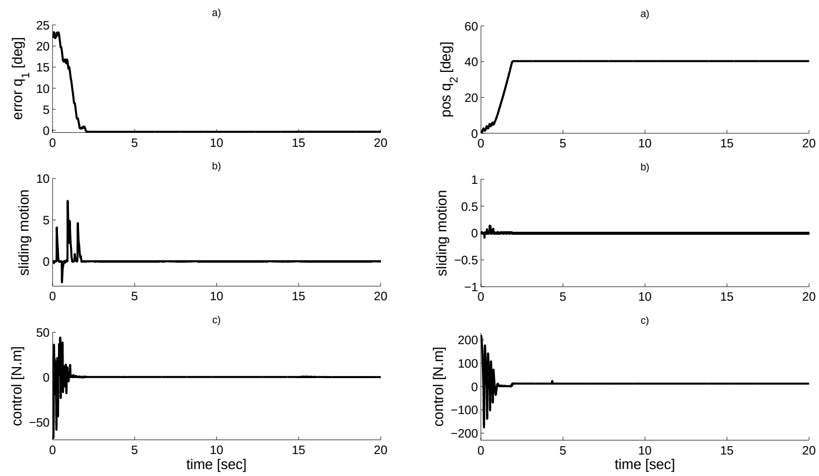

Fig. 3. Values for the joints q_1 and q_2.

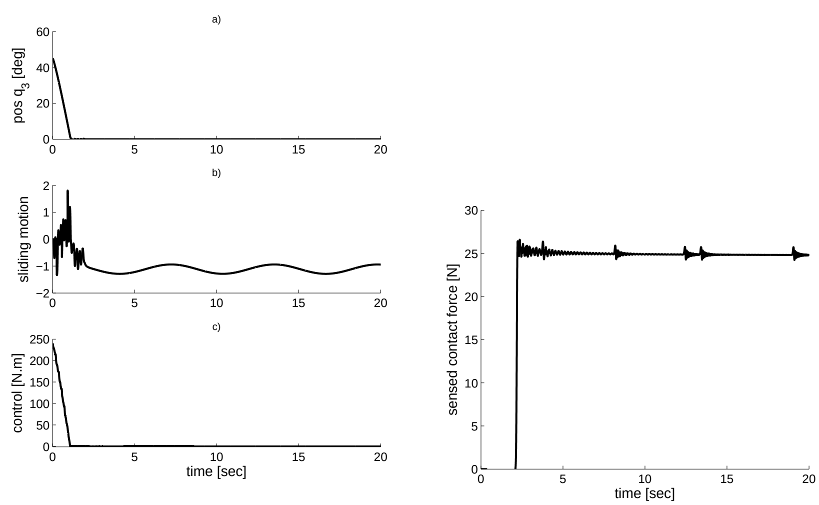

Fig. 4. Values for the joint q_3 and the sensed contact force on the end-effector.

constant value $N_i > 0$, with $i = 1, 2, 3$, whenever this inequality is not satisfied the controller will attenuate the disturbances and dead zone model discrepancies, but it is enough to increase the elements of the matrix diag$\{\beta\}$ in order to fulfill the inequality and turn it into an asymptotically stable system.

The experimental platform consists in a robot manipulator of three degrees-of-freedom named Pegasus which operates under constrained conditions, and it has a force sensor mounted at the end-effector. This platform has a transmission made of chains and gears, which presents a considerable backlash phenomenon in each joint.

The controller gains were chosen heuristically, therefore, it is possible that another gain values yield better results. Effectiveness of the design procedure has been supported by experiments in the platform of the Pegasus robot configured to have a position constraint.

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